SITE-SPECIFIC ACCURACY OF DIGITIZED PROPERTY MAPS

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BIOGRAPHICAL SKETCH

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ABSTRACT

The Westport Land Records Project, at the University of Wisconsin, is a multidisciplinary research project devoted to the improvement of land information. One of the goals of the project is to investigate the feasibility of merging various source map data by fitting digital representations of maps to ground surveyed control points. One set of source maps used in the study are the 1 inch = 400 ft. (1:4800) sectionalized property maps of Dane County, Wisconsin, which are currently being used for tax assessing, zoning, planning, permit granting, and floodplain insurance purposes. These maps were originally produced in the 1930's and updated from time to time. They are known to contain gross errors. Some of these maps have been digitized and adjusted to surveyed ground points. A series of three two-dimensional transformation models have been investigated: 1) conformal Helmert, 2) affine, and 3) projective. In order to assess the appropriateness of these models and the site-specific accuracy of the computed coordinates, adjusted map positions have been compared to those of field monumented points whose positions were determined by ground survey but withheld from the adjustment.

INTRODUCTION

The Westport Land Records Project, at the University of Wisconsin-Madison, is a multidisciplinary research effort in which the University's Department of Landscape Architecture, Department of Civil and Environmental Engineering and School of Natural Resources, the U.S. Department of Agriculture, the Dane County Regional Planning Commission, and the Wisconsin State Cartographer's Office are participating (Moyer, et al., 1981). One of the activities of the project is the investigation of the integration of resource, planimetric, and property maps by adjusting digitized map coordinates to surveyed ground control (Mills, 1982). The base maps for this investigation are the Dane County sectionalized property maps at a scale of 1:4800. These maps are currently used by the county for tax assessing, zoning, planning, permit granting, and floodplain insurance purposes. The maps, on linen, were originally produced, in the 1930's, from record survey information and legal descriptions. They are known to contain gross errors mainly attributable to a lack of spatial data on property boundaries and to drafting. They are manually updated from time to time as survey information becomes available.

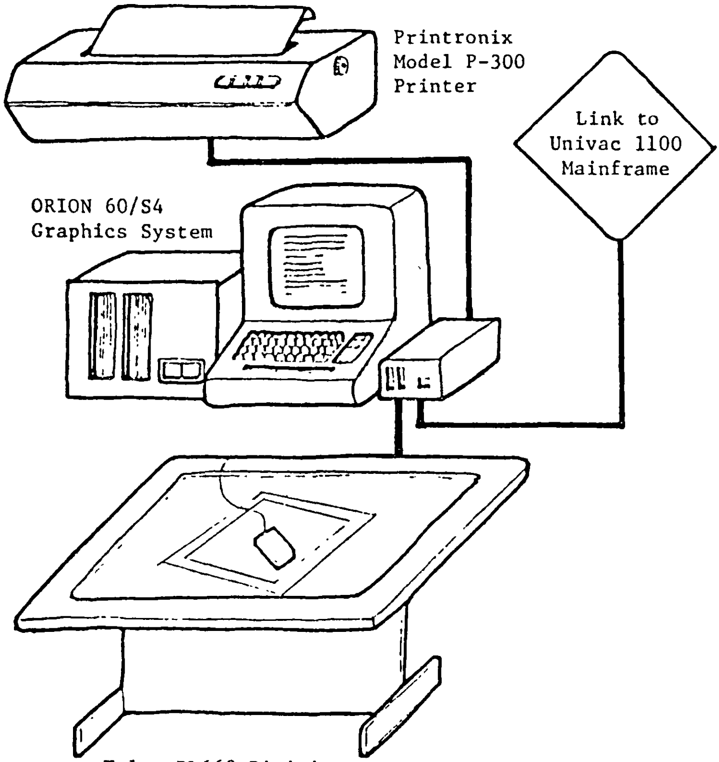
All of the section and quarter section corners in Westport Township, Dane County, have been remonumented in recent years under a county program. State plane coordinates (Wisconsin South Zone) on a number of these corners have been determined by ground survey. These coordinates are used, on the Westport Project, to control the transformation of map coordinates, obtained by digitizing the property maps, into the state plane system. Three different transformation models, with varying amounts of control, have been used.

Prior to the work reported herein, questions arose as to the appropriateness of the selected mathematical transformation models and as to the accuracy of the final transformed coordinates on property corners. The Northeast Quarter of Section 21, Township 8 North, Range 9 East of the 4th Principal Meridian was selected as a test site. The intent was to establish state plane coordinates on a number of property corners by ground survey and to compare these coordinates to those obtained by digitizing the property maps. This site was selected because it contains four residential subdivisions laid out within the past seventeen years. It was anticipated that a number of original property corner monuments would be recoverable in the proximity of existing horizontal control. Also, state plane coordinates of ten property corners along the east line of the Northeast Quarter of Section 21 had already been established in a prior study (Crossfield and Mezera, 1981).

DATA ACQUISITION

Map Digitization

Paper reproductions of the original linen property maps were digitized on a Talos BL660 Digitizer. The Digitizer has a bombsight cursor and a resolution of 0.03 mm. The standard deviation in a single observation on the digitizer was determined to be $\pm$ 0.18 mm from repeated measurements. At a scale of 1:4800, this results in a ground positional error of $\pm$ 0.85 meters. Figure 1 (taken from Mills (1982)) illustrates the full configuration of the equipment used for data capture. The ORION 60/54 Graphics System serves as a display and storage device for digited coordinates. Data may be dumped directly to the printer or transmitted to the university's UNIVAC 1100 mainframe computer which is used for hard-copy graphics.



Hardware Configuration for Map Digitization

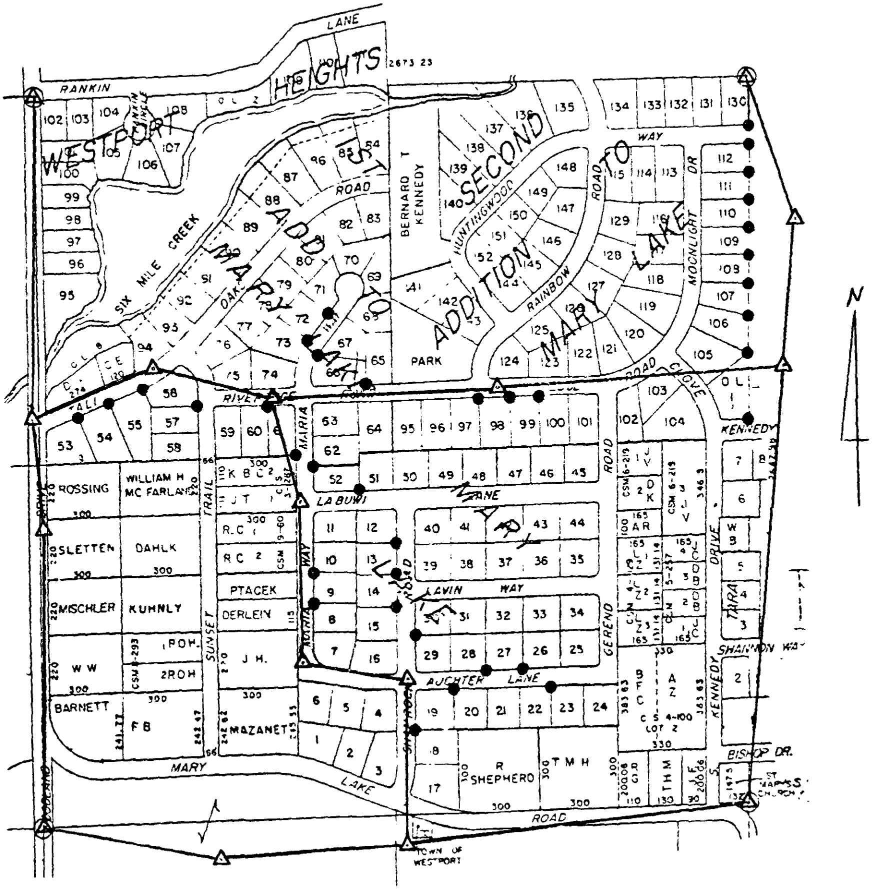
Figure 1.

The digitized map coordinates are subject to three sources of error: 1) errors in the original property maps, 2) errors in the reproduction process, and 3) errors in the digitizer measurements.

Ground Survey

The ground survey entailed two days of point recovery and two days of measurement. Successive property corners were recovered at random locations in three of the subdivisions. Only solid, undisturbed pins were used. Thirty-six pins were eventually unearthed and surveyed. Traverse was run connecting the recovered property corners to all four of the corners of the Northeast Quarter of Section 21. State plane coordinates of these quarter section corners were known. Figure 2 illustrates the configuration of the traverse network and the property corners.

The actual measurements were done with a Zeiss Elta 2S total station surveying instrument which facilitated electronic angle, distance, and elevation difference measurement plus automatic reduction and recording of surveyed data. Based



Traverse Network and Surveyed Property Corners

Figure 2.

strictly on surveying experience, this instrument resulted in somewhere between a 30-50% savings in measuring time. Also, the instrument allowed the entire survey to be performed with a two-man crew. The Elta is equipped with several additional computing capabilities such as free stationing and coordinate recall and computation. Were we to return to Westport, it would be possible to recall traverse point coordinates and coordinates for other points to be found resulting in the machine computing and measuring to search areas.

DATA REDUCTION

Map Digitization

The digitized map coordinates of the property corners were run through a series of three transformations. The first transformation is a four-parameter, conformal, Helmert of the form

$$N_T = N_{CG} + \lambda D^* \cos(Az+\phi)$$
$$E_T = E_{CG} + \lambda D^* \sin(Az+\phi) \quad (1)$$

where

N_T, E_T are the transformed coordinates,

N_{CG}, E_{CG} are the coordinates of the center of gravity of the control points in the state plane system,

λ is a scale factor,

ϕ is a rotation angle,

$D = \sqrt{(n-n_{CG})^2 + (e-e_{CG})^2}$

$Az = \tan^{-1}\left(\frac{e-e_{CG}}{n-n_{CG}}\right)$,

n,e are map coordinates to be transformed,

n_{CG}, e_{CG} are map coordinates of the center of gravity of the control points.

The second transformation is a six-parameter affine of the form

$$N_T = a_1 n + a_2 e + a_3$$
$$E_T = b_1 n + b_2 e + b_3, \quad (2)$$

where

$a_1, a_2, a_3, b_1, b_2, b_3$ are the affine coefficients,

N_T, E_T, n, e are as before.

The third transformation is an eight-parameter projective of the form

$$N_T = \frac{a_1 n + a_2 e + a_3}{c_1 n + c_2 e + 1}$$
$$E_T = \frac{b_1 n + b_2 e + b_3}{c_1 n + c_2 e + 1} \quad (3)$$

where

$a_1, a_2, a_3, b_1, b_2, b_3$ are the projective coefficients,

N_T, E_T, n, e are as before.

The three transformations require a minimum of two, three, and four control points, respecitvely, in order to solve for their unknowns. Any redundancy results in a least squares fitting of the model to the control. After a least squares fit, transformed coordinates may be refined by distributing the residuals at the control points using an inverse weighted distance function of the form

$$\Delta N_j = \frac{\sum_{i=1}^{n} v_{N_i} * P_i}{\sum_{i=1}^{n} P_i} \qquad \Delta E_j = \frac{\sum_{i=1}^{n} v_{E_i} * P_i}{\sum_{i=1}^{n} P_i}$$

where

$\Delta N, \Delta E$ are to be added to the transformed coordinates of point j,

V_{N_i}, V_{E_i} are the residuals in the transformed coordinates at the ith control point,

$P_i = \frac{1}{S_i}$ for $S_i > 1$,

$P_i = 1$ for $S_i \leq 1$,

$S_i = \sqrt{(N_j - N_i)^2 + (E_j - E_i)^2}$,

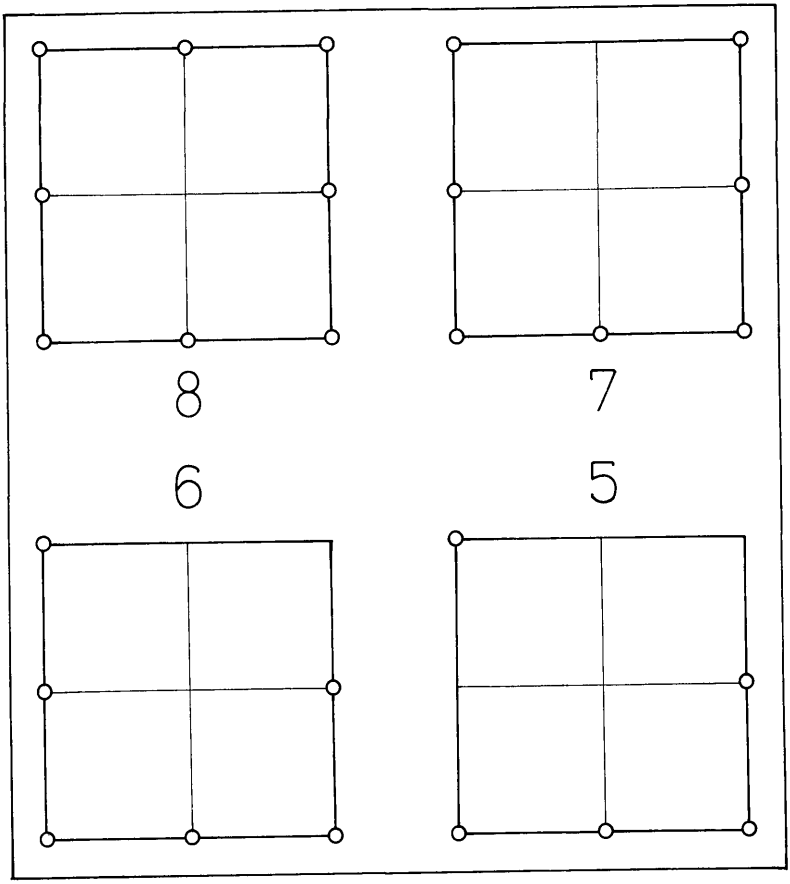
N_j, E_j are the transformed coordinates of point j,

N_i, E_i are the transformed coordinates of the ith control point.

Each of the transformations were performed for the four different configurations of control in Figure 3. Initially all eight section and quarter section corners were used as control in the transformations. The successive configurations of seven, six, and five control points were determined by discarding the control point with the largest residual in the conformal Helmert transformation of the immediately previous configuration.

Ground Survey

The measured traverse data was adjusted by the method of least squares. The network had nine degrees of freedom. The largest semi-major standard error ellipse axis was 15 mm and the largest semi-minor standard error ellipse axis was 11 mm. Result of the χ^2 test on the variance of unit weight at 95% confidence were as follows:



Control Configuration for Map Transformations

Figure 3.

$$H_o: \ \sigma^2 = \sigma_o^2 \ \text{vs.} \ H_1: \ \sigma^2 \neq \sigma_o^2$$

$$\frac{df\ \hat{\sigma}_o^2}{\chi^2_{1-\frac{\alpha}{2},df}} < \sigma_o^2 < \frac{df\hat{\sigma}_o^2}{\chi^2_{\frac{\alpha}{2},df}}$$

where

σ^2 = variance of unit weight,

σ_o^2 = a priori estimate of σ^2 = 1,

$\hat{\sigma}_o^2$ = a posteriori estimate of σ^2 = 1.46,

df = degrees of freedom = 9,

$\alpha = 5\%,$

$\chi^2_{.025,9} = 2.70,$

$\chi^2_{.975,9} = 19.02,$

and $1.0 \leq 1.0 \leq 7.1$ (accept H_o).

The coordinates of the property corners were computed as sideshots from the adjusted traverse coordinates.

ACCURACY ANALYSIS

The approach used in analyzing the accuracy of the transformed property maps is that proposed by the Specifications and Standards Committee of ASP as presented in Merchant (1982). The Circular Map Accuracy Standard at the scale of the source maps (1 inch = 400 ft., 1:4800) requires that 90% of the well-defined points be in error by less than

2.6 m (8.6 ft.) for Class 1 maps,
5.2 m (17. ft.) for Class 2 maps,
7.8 m (26. ft.) for Class 3 maps.

These correspond to standard errors (1σ) of

1.2 m for Class 1 maps,
2.4 m for Class 2 maps,
3.6 m for Class 3 maps.

Compliance with these standards is examined by testing for bias (student's t) and precision (χ^2) in both the northing (Y) and easting (X) directions of the maps at a 95% confidence level. Any given map passes the test if

$$|t_N| \leq t_{n-1,\alpha} \quad , \quad |t_E| \leq t_{n-1,\alpha} \quad ,$$

$$|\chi^2_N| \leq \chi^2_{n-1,\alpha} \quad , \quad \text{and } |\chi^2_E| \leq \chi^2_{n-1,\alpha}$$

where

$$t_N = \frac{1}{S_N}(\overline{\Delta N})\sqrt{n} \quad , \quad t_E = \frac{1}{S_E}(\overline{\Delta E})\sqrt{n} \quad ,$$

$$\chi^2_N = (\frac{n-1}{\sigma^2})S^2_N \quad , \quad \chi^2_E = (\frac{n-1}{\sigma^2})S^2_E \quad , \qquad (5)$$

$\overline{\Delta N}, \overline{\Delta E}$ are the sample means of the discrepancies ($\Delta N, \Delta E$) in the northings and eastings at the check points,

S_N, S_E are the corresponding sample standard deviations,

n = number of check points,

σ is the standard error as stated above for Class 1, 2, and 3 maps.

Table 1.
Coordinate Discrepancies for Conformal Helmert Transformation Using 8 Control Points

	Undistributed Residuals		Distributed Residuals	
Point #	ΔN(meters)	ΔE(meters)	ΔN(meters)	ΔE(meters)
1	-5.08	-0.30	-3.19	-0.31
2	-5.20	-0.28	-3.30	-0.27
3	-5.23	1.13	-3.31	1.19
4	-5.08	-0.52	-3.09	-0.35
5	-4.28	-0.74	-1.99	-0.33
6	0.39	-2.76	2.79	-2.22
7	-0.83	2.79	1.71	3.26
8	-1.18	2.40	1.41	2.89
9	-4.41	1.48	-1.96	1.90
10	-4.16	1.91	-1.94	2.19
11	-1.66	-3.25	0.96	-2.57
12	-1.10	-3.93	1.42	-3.22
13	-0.33	-2.02	2.05	-1.28
14	-2.25	-1.27	-0.25	-1.02
15	-1.68	-1.27	0.26	-1.05
16	-3.81	2.30	-2.12	2.30
17	-1.60	0.50	-0.21	0.40
18	-0.30	0.80	0.98	0.63
19	-1.65	4.89	-0.25	4.57
20	-0.68	4.77	0.81	4.54
21	-3.40	1.35	-2.17	0.81
22	-1.10	6.55	0.07	5.66
23	-1.11	5.57	0.09	4.54
24	-1.03	2.84	0.16	1.62
25	-1.08	2.99	0.08	1.90
26	-1.75	2.12	-0.63	1.30
27	-3.87	4.59	-2.88	3.81
28	-4.51	-2.02	-1.27	-0.35
29	-8.28	-2.20	-5.21	-0.74
30	-4.04	-1.96	-1.20	-0.76
31	-4.88	-1.44	-2.21	-0.45
32	-4.20	-1.39	-1.70	-0.60
33	-3.84	-1.22	-1.48	-0.61
34	-3.62	-1.08	-1.39	-0.64
35	-4.37	-0.10	-2.41	-0.06
36	-5.02	0.32	-3.26	0.03

Also,

$$t_{35,.05} = 1.690 \quad \text{and} \quad \chi^2_{35,.05} = 49.76. \tag{6}$$

As indicated by equations (5), all three classes of map must pass the same test for bias. However, because of the appearance of the standard error in the sample χ^2_N and χ^2_E statistics, precision requirements are not as stringent for map classes of lower accuracy as they are for map classes of higher accuracy.

Table 1 contains the coordinate discrepancies at the thirty-six check points for the conformal Helmert transformation using eight control points. The first two columns are data derived prior to refinement by residual distribution. Since the transformation contains only two translations, one rotation, and one scale factor, these discrepancies are indicative of the error in the digitized map at ground scale. The second two columns are data derived after the distribution of residuals using equations (4). There is a downward trend in the magnitude of the residuals after the coordinate refinement.

Table 2 contains the sample means and standard deviations in the coordinate discrepancies at all check points for each of the control configurations, versus each of the transformations with distributed residuals and without. The magnitude of the mean residuals tends to increase as the number of control points decreases. However, there is no corresponding trend in the standard deviations.

The affine transformation should account for some of the systematic effects of paper stretching during map reproduction. In fourteen out of sixteen cases the mean residuals for the affine transformation are smaller than those for the conformal. The projective transformation should account for differential scale factors and for misplotting of the section and quarter section corners. In thirteen out of sixteen cases the mean residuals for the affine transformation are smaller than those for the conformal. In nearly all cases the affine and projective transformation have smaller standard deviations than the conformal.

Table 3 contains absolute values of the sample t statistics and the results of the tests when they are compared to the theoretical t statistic in equations (6). The number of failures of the test for bias increases as the number of control points decreases. When eight control points are used, the affine and projective transformations, in all cases, have less bias than the conformal. However, there is no clear choice to be made between the affine transformation and the projective. As can be seen, coordinate refinement, by distribution of the residuals, can actually introduce additional bias in the map. None of the maps can be considered bias-free because, in all cases, the t test failed in one or both of the N and E directions.

From equations (5) and (6) and the given acceptable standard errors for the three map classes, the maximum allowable sample standard deviations in either direction are:

$$s_{max} = \frac{(\chi^2_{n-1,\alpha})\sigma^2}{n-1} = \frac{49.76*1.2^2}{35} = 1.43 \text{ m (Class 1)},$$

$$s_{max} = \frac{49.76*2.4^2}{35} = 2.86 \text{ m (Class 2)},$$

$$s_{max} = \frac{49.76*3.6^2}{35} = 4.29 \text{ m (Class 3)}.$$

Table 2.
Sample Means and Standard Deviations in Coordinate Discrepancies
(Units are Meters)

Number of Control Points		Conformal				Affine				Projective			
		Undistributed Residuals		Distributed Residuals		Undistributed Residuals		Distributed Residuals		Undistributed Residuals		Distributed Residuals	
		ΔN	ΔE	ΔN	ΔE	ΔN	ΔE	ΔN	ΔE	ΔN	ΔE	ΔN	ΔE
8	Mean	-2.95	0.59	-0.96	0.74	0.45	0.35	0.56	0.57	-0.12	0.73	0.03	0.71
	Std.Dev.	1.96	2.62	1.84	2.14	1.45	2.30	1.68	2.06	1.51	2.04	1.79	1.99
7	Mean	-5.04	-0.40	-3.17	0.11	-1.69	0.78	-1.58	1.06	-2.05	1.06	-1.94	0.79
	Std.Dev.	2.06	2.67	1.55	2.10	1.50	2.25	1.47	2.04	1.45	2.08	1.45	3.00
6	Mean	-7.76	-2.03	-6.59	-1.94	-2.31	-0.91	-2.24	-0.72	-2.36	-0.20	-2.23	-0.09
	Std.Dev.	2.34	2.73	2.38	2.69	1.56	2.49	1.55	2.52	1.63	2.45	1.62	2.46
5	Mean	-6.63	-2.54	-5.37	-2.46	-2.20	-1.27	-2.09	-1.12	-2.71	-0.92	-2.53	-0.79
	Std.Dev.	2.20	2.66	2.26	2.62	1.59	2.42	1.59	2.43	1.77	2.56	1.77	2.57

Table 3.
Results of the Tests for Bias in Northing and Easting

Number of Control Points	Conformal				Affine				Projective			
	Undistributed Residuals		Distributed Residuals		Undistributed Residuals		Distributed Residuals		Undistributed Residuals		Distributed Residuals	
	$\lvert t_N \rvert$	$\lvert t_E \rvert$	$\lvert t_N \rvert$	$\lvert t_E \rvert$	$\lvert t_N \rvert$	$\lvert t_E \rvert$	$\lvert t_N \rvert$	$\lvert t_E \rvert$	$\lvert t_N \rvert$	$\lvert t_E \rvert$	$\lvert t_N \rvert$	$\lvert t_E \rvert$
8	9.03 Fail	1.35 Pass	3.13 Fail	2.07 Fail	1.86 Fail	0.91 Pass	2.00 Fail	1.66 Pass	0.48 Pass	2.15 Fail	0.10 Pass	2.14 Fail
7	14.7 Fail	0.90 Pass	12.3 Fail	0.31 Pass	6.76 Fail	2.08 Fail	6.45 Fail	3.12 Fail	8.48 Fail	3.06 Fail	8.03 Fail	1.58 Pass
6	19.9 Fail	4.46 Fail	16.6 Fail	4.33 Fail	8.88 Fail	2.19 Fail	8.67 Fail	1.71 Fail	8.69 Fail	0.49 Pass	8.26 Fail	0.22 Pass
5	18.1 Fail	5.73 Fail	14.3 Fail	5.63 Fail	8.30 Fail	3.15 Fail	7.89 Fail	2.77 Fail	9.19 Fail	2.16 Fail	8.58 Fail	1.84 Fail

Using these criteria, and the values in Table 2, all of the maps except one pass the Class 2 precision test. The remaining map passes the Class 3 precision test.

CONCLUSIONS

It was our original intent to perform what we believed was one of the only studies of the relationship between actual property corners with legal significance and their representation on maps which have undergone mathematical manipulation. We had hoped to thereby ascertain the usefulness of our digital map product.

As can be seen from the tabulated results, none of our digital map products satisfies the large scale map accuracy standards as proposed by ASP. Most passed the test for Class 2 precision, but all failed the test for bias. We did note an improvement in the computed positions due to the mathematical transformation.

More importantly however, this study had made obvious the intuitive idea that no amount of mathematical manipulation of an inherently poor source document can result in an accurate final product. However, we do not feel that our effort has been entirely fruitless. The digital map products are certainly more versatile than their linen counterparts, and may be useful for planning and zoning purposes.

The further implication that existing land records cannot be used to produce accurate property maps is not substantiated by this work. Further investigation is required.

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REFERENCES

Crossfield, J.K. and D.F. Mezera, (1981), "The Westport Section Line Accuracy Study," Proceedings of the ACSM Fall Technical Meeting, September.

Merchant, D.C., (1982), "Spatial Accuracy Standards for Large-Scale Line Maps," Proceedings of the ACSM 42nd Annual Meeting, March.

Mills, G.L., (1982), "An Automated Approach to the Overlay of Wisconsin Wetlands Inventory Data with Local Property Maps," Masters Thesis, University of Wisconsin-Madison.

Moyer, D., Portner, J., and D.F. Mezera, (1981), "Overview of a Survey-Based System for Improving Data Compatability in Land Record Systems," Proceedings of the 19th Annual Conference of the Urban and Regional Information Systems Association, August.